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ABSTRACT

The calculator is used to explore the distributive property of a unary operator over a binary operator. This is an extension of an earlier paper. The emphasis is on student discovery of the property for various combinations of the operators. Instructional material contained in the report include a reference sheet of questions for potential investigation with an accompanying space to record a general calculator algorithm and a record sheet. (MP)

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Project Paper 77-6

SOME MONADIC/DIADIC COMBOS

by

J. F. Weaver

Report from the
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Sincere appreciation is expressed to Leonard Rush (Principal), Ms Jo Marshall (Learning Coordinator), Jack Novick (Teacher) and his seventh-grade mathematics students for their interest and cooperation in making possible within the Thomas Jefferson Middle School (Madison, WI) the calculator explorations which included this instructional module or unit.

Acknowledgment also is made of the various ways in which Charles Mitchell and Glen Blume, Ph.D. candidates in mathematics education, have assisted with the calculator explorations.

This module or unit is an extension of Project Paper 77-5, *A monadic module, alias A unary unit* (Weaver, 1977a). It is assumed that students have worked through that material as background for this present module or unit. Such background will not be repeated here.

The explorations to be made are associated with a set of questions that may be expressed *algebraically* in the following form:

Under what condition(s) for a and b is it true that $d = c$ when:

1. $(a + b)^2 = c$ and $a^2 + b^2 = d$?

2. $(a - b)^2 = c$ and $a^2 - b^2 = d$?

3. $(a \times b)^2 = c$ and $a^2 \times b^2 = d$?

4. $(a \div b)^2 = c$ and $a^2 \div b^2 = d$?

5. $\sqrt{a + b} = c$ and $\sqrt{a} + \sqrt{b} = d$?

6. $\sqrt{a - b} = c$ and $\sqrt{a} - \sqrt{b} = d$?

7. $\sqrt{a \times b} = c$ and $\sqrt{a} \times \sqrt{b} = d$?

8. $\sqrt{a \div b} = c$ and $\sqrt{a} \div \sqrt{b} = d$?

9. $\frac{1}{a + b} = c$ and $\frac{1}{a} + \frac{1}{b} = d$ [or $(a + b)^{-1} = c$ and $a^{-1} + b^{-1} = d$] ?

#10. $\frac{1}{a - b} = c$ and $\frac{1}{a} - \frac{1}{b} = d$ [or $(a - b)^{-1} = c$ and $a^{-1} - b^{-1} = d$] ?

#11. $\frac{1}{a \times b} = c$ and $\frac{1}{a} \times \frac{1}{b} = d$ [or $(a \times b)^{-1} = c$ and $a^{-1} \times b^{-1} = d$] ?

#12. $\frac{1}{a \div b} = c$ and $\frac{1}{a} \div \frac{1}{b} = d$ [or $(a \div b)^{-1} = c$ and $a^{-1} \div b^{-1} = d$] ?

However:

In terms of the calculator-influenced unary-operator notation of Project Paper 77-5, these same questions may be posed in a different way:

Under what condition(s) for a and b is it true that $d = c$ when:

1. $(a + b)\boxed{x^2} = c$ and $(a\boxed{x^2}) + (b\boxed{x^2}) = d$?

2. $(a - b)\boxed{x^2} = c$ and $(a\boxed{x^2}) - (b\boxed{x^2}) = d$?

3. $(a \times b)\boxed{x^2} = c$ and $(a\boxed{x^2}) \times (b\boxed{x^2}) = d$?

4. $(a \div b)\boxed{x^2} = c$ and $(a\boxed{x^2}) \div (b\boxed{x^2}) = d$?

5. $(a + b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) + (b\boxed{\sqrt{\ }}) = d$?

6. $(a - b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) - (b\boxed{\sqrt{\ }}) = d$?

7. $(a \times b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) \times (b\boxed{\sqrt{\ }}) = d$?

8. $(a \div b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) \div (b\boxed{\sqrt{\ }}) = d$?

9. $(a + b)\boxed{1/x} = c$ and $(a\boxed{1/x}) + (b\boxed{1/x}) = d$?

#10. $(a - b)\boxed{1/x} = c$ and $(a\boxed{1/x}) - (b\boxed{1/x}) = d$?

#11. $(a \times b)\boxed{1/x} = c$ and $(a\boxed{1/x}) \times (b\boxed{1/x}) = d$?

#12. $(a \div b)\boxed{1/x} = c$ and $(a\boxed{1/x}) \div (b\boxed{1/x}) = d$?

The preceding format is the one used with a class of accelerated seventh-grade students (who will study "first-year algebra" in grade 8 rather than in grade 9) in connection with their calculator explorations.[1]

In contrast to the "mishmash" of conventional algebraic notation, the consistency of the calculator-based notation makes it much more evident to students that all questions are of a common form involving sentences

$$(a \circ b)\boxed{\nabla} = c \text{ and } (a\boxed{\nabla}) \circ (b\boxed{\nabla}) = d$$

and that a question of *distributivity*--of ∇ over $\circ$ --is an overriding one throughout the instructional explorations.

Instructional Materials

In addition to a suitable calculator, each student should have the following:

1.0 - One copy of the REFERENCE SHEET of questions for potential investigation (page 4 of this manuscript), with accompanying space to record a *general* Calculator Algorithm that covers the set of 12 questions. Such an algorithm is illustrated on page 5 (this manuscript) for the National Semiconductor "Mathematician" 4510 RPN calculator used by students in the accelerated seventh-grade class.

2.0 - For each question investigated by a student:

2.1 - Another copy of the REFERENCE SHEET on which the student can record an algorithm particularized for the question being investigated. This is illustrated on page 6 (this manuscript) for question #10.

2.2 - A copy of the RECORD SHEET (page 7, this manuscript), also to be particularized for the question being investigated--as illustrated by page 8 (this manuscript) for question #10. For each data line on a RECORD SHEET the student is to indicate whether $d = c$ or $d \neq c$.

Variation can be expected in the degree to which teacher guidance is helpful or necessary for students to reach the conclusions summarized on page 9 (this manuscript), with particular attention to operations and conditions for which $\boxed{\nabla}$ is distributive over $\boxed{\circ}$.

User Instructions and accompanying program are included on pages 10 and 11 (this manuscript) for persons who may wish to use an *EduCALC* 29GD, as I did, when working with students on this unit or module.

REFERENCE SHEET

$(a \circ b)\boxed{\nabla} = c$ and $(a\boxed{\nabla}) \circ (b\boxed{\nabla}) = d$

Under what condition(s) for a and b is it true that $d = c$ when:

1. $(a + b)\boxed{x^2} = c$ and $(a\boxed{x^2}) + (b\boxed{x^2}) = d$?

2. $(a - b)\boxed{x^2} = c$ and $(a\boxed{x^2}) - (b\boxed{x^2}) = d$?

3. $(a \times b)\boxed{x^2} = c$ and $(a\boxed{x^2}) \times (b\boxed{x^2}) = d$?

4. $(a \div b)\boxed{x^2} = c$ and $(a\boxed{x^2}) \div (b\boxed{x^2}) = d$?

5. $(a + b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) + (b\boxed{\sqrt{\ }}) = d$?

6. $(a - b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) - (b\boxed{\sqrt{\ }}) = d$?

7. $(a \times b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) \times (b\boxed{\sqrt{\ }}) = d$?

8. $(a \div b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) \div (b\boxed{\sqrt{\ }}) = d$?

9. $(a + b)\boxed{1/x} = c$ and $(a\boxed{1/x}) + (b\boxed{1/x}) = d$?

#10. $(a - b)\boxed{1/x} = c$ and $(a\boxed{1/x}) - (b\boxed{1/x}) = d$?

#11. $(a \times b)\boxed{1/x} = c$ and $(a\boxed{1/x}) \times (b\boxed{1/x}) = d$?

#12. $(a \div b)\boxed{1/x} = c$ and $(a\boxed{1/x}) \div (b\boxed{1/x}) = d$?

Calculator Algorithm

Step	Input	Key(s)	Output

REFERENCE SHEET

$(a \circ b)\,[\nabla] = c$ and $(a\,[\nabla]) \circ (b\,[\nabla]) = d$

Under what condition(s) for a and b is it true that $d = c$ when:

1. $(a + b)\,[x^2] = c$ and $(a\,[x^2]) + (b\,[x^2]) = d$?
2. $(a - b)\,[x^2] = c$ and $(a\,[x^2]) - (b\,[x^2]) = d$?
3. $(a \times b)\,[x^2] = c$ and $(a\,[x^2]) \times (b\,[x^2]) = d$?
4. $(a \div b)\,[x^2] = c$ and $(a\,[x^2]) \div (b\,[x^2]) = d$?
5. $(a + b)\,[\sqrt{\ }] = c$ and $(a\,[\sqrt{\ }]) + (b\,[\sqrt{\ }]) = d$?
6. $(a - b)\,[\sqrt{\ }] = c$ and $(a\,[\sqrt{\ }]) - (b\,[\sqrt{\ }]) = d$?
7. $(a \times b)\,[\sqrt{\ }] = c$ and $(a\,[\sqrt{\ }]) \times (b\,[\sqrt{\ }]) = d$?
8. $(a \div b)\,[\sqrt{\ }] = c$ and $(a\,[\sqrt{\ }]) \div (b\,[\sqrt{\ }]) = d$?
9. $(a + b)\,[1/x] = c$ and $(a\,[1/x]) + (b\,[1/x]) = d$?
#10. $(a - b)\,[1/x] = c$ and $(a\,[1/x]) - (b\,[1/x]) = d$?
#11. $(a \times b)\,[1/x] = c$ and $(a\,[1/x]) \times (b\,[1/x]) = d$?
#12. $(a \div b)\,[1/x] = c$ and $(a\,[1/x]) \div (b\,[1/x]) = d$?

General

Calculator Algorithm

Step	Input	Key(s)		Output
1	a	ENT	ENT	a
2	b	MS		b
3		$\circ$		$a \circ b$
4		∇		c
5		C↓	∇	$a\,[\nabla]$
6		MR	∇	$b\,[\nabla]$
7		$\circ$		d

REFERENCE SHEET

$(a \circ b)\boxed{\nabla} = c$ and $(a\boxed{\nabla}) \circ (b\boxed{\nabla}) = d$

Under what condition(s) for a and b is it true that $d = c$ when:

1. $(a + b)\boxed{x^2} = c$ and $(a\boxed{x^2}) + (b\boxed{x^2}) = d$?

2. $(a - b)\boxed{x^2} = c$ and $(a\boxed{x^2}) - (b\boxed{x^2}) = d$?

3. $(a \times b)\boxed{x^2} = c$ and $(a\boxed{x^2}) \times (b\boxed{x^2}) = d$?

4. $(a \div b)\boxed{x^2} = c$ and $(a\boxed{x^2}) \div (b\boxed{x^2}) = d$?

5. $(a + b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) + (b\boxed{\sqrt{\ }}) = d$?

6. $(a - b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) - (b\boxed{\sqrt{\ }}) = d$?

7. $(a \times b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) \times (b\boxed{\sqrt{\ }}) = d$?

8. $(a \div b)\boxed{\sqrt{\ }} = c$ and $(a\boxed{\sqrt{\ }}) \div (b\boxed{\sqrt{\ }}) = d$?

9. $(a + b)\boxed{1/x} = c$ and $(a\boxed{1/x}) + (b\boxed{1/x}) = d$?

#10. $(a - b)\boxed{1/x} = c$ and $(a\boxed{1/x}) - (b\boxed{1/x}) = d$?

#11. $(a \times b)\boxed{1/x} = c$ and $(a\boxed{1/x}) \times (b\boxed{1/x}) = d$?

#12. $(a \div b)\boxed{1/x} = c$ and $(a\boxed{1/x}) \div (b\boxed{1/x}) = d$?

Calculator Algorithm

for Question # 10

Step	Input	Key(s)		Output
1	a	ENT	ENT	a
2	b	MS		b
3		-		$a - b$
4		$1/x$		c
5		C↓	$1/x$	$a\boxed{1/x}$
6		MR	$1/x$	$b\boxed{1/x}$
7		-		d

RECORD SHEET

$(a \circ b)\,\boxed{\nabla} = c$ and $(a\,\boxed{\nabla}) \circ (b\,\boxed{\nabla}) = d$

#_____: $(a \quad b)\,\boxed{} = c$ and $(a\,\boxed{}) \quad (b\,\boxed{}) = d$

a	b	$(a \quad b)$	c	$a\,\boxed{}$	$b\,\boxed{}$	d	d ? c
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$
							$d \quad c$

RECORD SHEET

$(a \circ b)$ [▽] $= c$ and $(a$ [▽] $) \circ (b$ [▽] $) = d$

10 : $(a - b)$ [1/x] $= c$ and $(a$ [1/x] $) - (b$ [1/x] $) = d$

a	b	$(a - b)$	c	a [1/x]	b [1/x]	d	d ? c
							d c
							d c
							d c
							d c
							d c
							d c
							d c
							d c
							d c
							d c
							d c
							d c

c d	c d	c d
$((a + b)\boxed{x^2}) = (a\boxed{x^2} + b\boxed{x^2})$	$((a + b)\boxed{\surd}) = (a\boxed{\surd} + b\boxed{\surd})$	$((a + b)\boxed{1/x}) = (a\boxed{1/x} + b\boxed{1/x})$
[1] All a, $b = 0$ [2] All b, $a = 0$	[1] All $a \geq 0$, $b = 0$ [2] All $b \geq 0$, $a = 0$	
$((a - b)\boxed{x^2}) = (a\boxed{x^2} - b\boxed{x^2})$	$((a - b)\boxed{\surd}) = (a\boxed{\surd} - b\boxed{\surd})$	$((a - b)\boxed{1/x}) = (a\boxed{1/x} - b\boxed{1/x})$
[1] All a, $b = 0$ [2] All $a = b$	[1] All $a \geq 0$, $b = 0$ [2] All $(a \geq 0) = b$	
$((a \times b)\boxed{x^2}) = (a\boxed{x^2} \times b\boxed{x^2})$	$((a \times b)\boxed{\surd}) = (a\boxed{\surd} \times b\boxed{\surd})$	$((a \times b)\boxed{1/x}) = (a\boxed{1/x} \times b\boxed{1/x})$
[1] All a, b	[1] All $a \geq 0$, $b \geq 0$	[1] All $a \neq 0$, $b \neq 0$
$((a \div b)\boxed{x^2}) = (a\boxed{x^2} \div b\boxed{x^2})$	$((a \div b)\boxed{\surd}) = (a\boxed{\surd} \div b\boxed{\surd})$	$((a \div b)\boxed{1/x}) = (a\boxed{1/x} \div b\boxed{1/x})$
[1] All a, $b \neq 0$	[1] All $a \geq 0$, $b > 0$	[1] All $a \neq 0$, $b \neq 0$

1. For each of the preceding conditions, a and b are rational numbers.

EduCALC 29GD User Instructions

STEP	INSTRUCTIONS	INPUT DATA/UNITS	Key(s)		OUTPUT DATA/UNITS
1	Set display: p places:		f FIX	p	
	(May be changed any time)				
2	Set program pointer:		g	RTN	
3	Do ONE of these:				
	If unary op. is x^2:	1	STO	9	
	If unary op. is $\sqrt{x}$:	2	STO	9	
	If unary op. is $1/x$:	3	STO	9	
4	Do ONE of these:				
	If binary op. is + :	5	STO	0	
	If binary op. is - :	6	STO	0	
	If binary op. is × :	7	STO	0	
	If binary op. is ÷ :	8	STO	0	
5	Iterate steps 6 thru 10				
	for each a and b.				
6	Input a, b; calc. $a \circ b$:	a	ENTER↑		
		b	R/S		$a \circ b$
7	Calculate c:		R/S		c
8	Calculate a [▽] :		R/S		a [▽]
9	Calculate b [▽] :		R/S		b [▽]
10	Calculate d:		R/S		d

NOTE: Step 3 or/and Step 4 may be changed at the end of any iteration of Steps 6 thru 10 without disrupting program execution. If the display gives an Error message at any time, steps 2, 3, and 4 must be re-executed before continuing.

EduCALC GD29 Program

Key(s)	Line	Code
g LBL 0	01	15 13 00
STO 2	02	23 02
$x \rightleftharpoons y$	03	21
STO 1	04	23 01
$x \rightleftharpoons y$	05	21
GSB i	06	12 22
R/S	07	74
GSB 9	08	12 09
R/S	09	74
RCL 1	10	24 01
GSB i	11	12 22
R/S	12	74
RCL 2	13	24 02
GSB i	14	12 22
R/S	15	74
g LBL 9	16	15 13 09
RCL 9	17	24 09
RCL 0	18	24 00
STO 9	19	23 09
R↓	20	22
STO 0	21	23 00
R↓	22	22
GSB i	23	12 22

Key(s)	Line	Code
g RTN	24	15 12
GTO 0	25	13 00
g LBL 1	26	15 13 01
g x^2	27	15 63
g RTN	28	15 12
g LBL 2	29	15 13 02
f $\sqrt{x}$	30	14 63
g RTN	31	15 12
g LBL 3	32	15 13 03
g $1/x$	33	15 74
g RTN	34	15 12
g LBL 5	35	15 13 05
+	36	51
g RTN	37	15 12
g LBL 6	38	15 13 06
-	39	41
g RTN	40	15 12
g LBL 7	41	15 13 07
×	42	61
g RTN	43	15 12
g LBL 8	44	15 13 08
÷	45	71
g RTN	46	15 12

References

Weaver, J. F. *A monadic module, alias A unary unit* (Project Paper 77-5, Studies in Mathematics Work Group). Madison: Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin-Madison, December 1977. (a)

Weaver, J. F. *Calculators and unary operators* (Project Paper 77-7, Studies in Mathematics Work Group). Madison: Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin-Madison, December 1977. (b)

Footnote

[1] This material is derived from the writer's fourth year of calculator exploration and investigation, over a range of grade levels, as a Professor of Mathematics Education and Faculty Associate, Wisconsin Research and Development Center for Cognitive Learning--supported in part as a research and development center by funds from the National Institute of Education, Department of Health, Education, and Welfare (Center Contract No. NE-C-00-3-0065). The opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education and no official endorsement by that agency should be inferred.